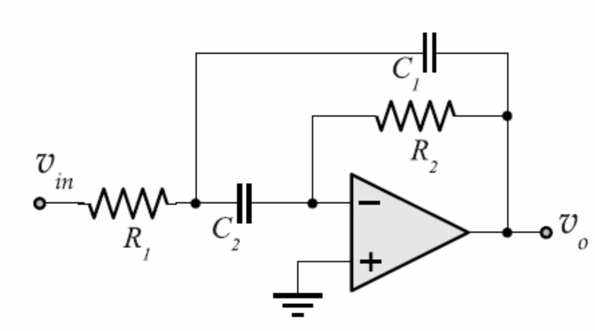
The MFBP Filter



The schematic above is called a "Multiple Feedback Bandpass" (MFBP) active filter. It provides one possible method of using a single op-amp to implement the generic 2nd order bandpass transfer function given by:

$$H(s) = \frac{K\left(\frac{\omega_p}{Q_p}\right)s}{s^2 + \left(\frac{\omega_p}{Q_p}\right)s + \omega_p^2} \qquad \text{(Eq.1)}$$

The values ω_p and Q_p are called the "pole frequency" and the "pole Q" respectively. The bandpass characteristic $|H(j\omega)|$ is (roughly) centered at $\omega = \omega_p$, where the filter gain is $|H(j\omega)| = |K|$. Small values of Q_p result in very broad frequency responses about the center frequency, while larger values give sharp filter characteristics. As Q_p becomes large, the value of Q_p reflects the ratio of the center-frequency to the 3-dB bandwidth. For example, a 20 kHz 2nd order bandpass filter with 3-dB bandwidth 1 kHz would require $\omega_p \cong 2\pi(20\text{x}10^3)$ and $Q_p \cong 20$.

Higher order filters can be formed by cascading MFBP sections, where each section is dedicated to implementing a complex-conjugate pair of poles. For conjugate-pole locations of $p_i = \sigma \pm j\omega_0$, the resulting values of ω_p and Q_p are:

$$\omega_p = \sqrt{\sigma^2 + \omega_0^2}$$

$$Q_p = \frac{1}{2}\sqrt{1 + \frac{\omega_0^2}{\sigma^2}}$$

Assuming ideal op-amp characteristics, the transfer function of the above active filter can shown to be:

$$\frac{v_o(s)}{v_{in}(s)} = \frac{-\left(\frac{1}{R_1C_1}\right)s}{s^2 + \left(\frac{1}{R_2C_1} + \frac{1}{R_2C_2}\right)s + \frac{1}{R_1R_2C_1C_2}} \qquad \text{(Eq.2)}$$

To ease component matching requirements, most designers select a single convenient capacitor value $C = C_1 = C_2$ (see mini tutorial #220 from Analog Devices, AD MT-220). The remaining components are selected to obtain the desired values of ω_p, Q_p and K. You'll find that for this circuit, the gain cannot be set independently from Q_p. The gain can be lowered by using a resistor divider on the input, as in the AD MT-220, but increasing the gain would require a second amplifier stage.

From the above expressions:

$$H(s) = \frac{K\left(\frac{\omega_p}{Q_p}\right)s}{s^2 + \left(\frac{\omega_p}{Q_p}\right)s + \omega_p^2} = \frac{-\left(\frac{1}{R_1C_1}\right)s}{s^2 + \left(\frac{1}{R_2C_1} + \frac{1}{R_2C_2}\right)s + \frac{1}{R_1R_2C_1C_2}} \quad \text{(Eq.3)}$$

The following three relations are obtained:

$$K\left(\frac{\omega_p}{Q_p}\right)s = -\left(\frac{1}{R_1C_1}\right)s \quad \text{(Eq.4)}$$

$$\frac{\omega_p}{Q_p} = \frac{1}{R_2C_1} + \frac{1}{R_2C_2} \quad \text{(Eq.5)}$$

$$\omega_p^2 = \frac{1}{R_1R_2C_1C_2} \quad \text{(Eq.6)}$$

Usually $C_1 = C_2$ is chosen to simplify the design. Then, from Eq.4 and Eq.5 we have:

$$K\left(\frac{1}{R_2C_1} + \frac{1}{R_2C_2}\right) = -\frac{1}{R_1C_1}$$

$$K\left(\frac{2}{R_2}\right) = -\frac{1}{R_1}$$

From which we obtain:

$$K = -\frac{R_2}{2R_1} \quad \text{(Eq. 7)}$$

If the gain of the filter is known, then choosing one resistor value will set the second resistor's value. We can further manipulate the equations to find a relation between the gain and the quality factor:

$$\omega_p = 2\pi f_p = \frac{1}{\sqrt{R_1R_2}C_1}$$

$$\left|\frac{K}{Q}\right| 2\pi f_p = \frac{1}{R_1C_1}$$

$$\left(\frac{1}{R_2C_1} + \frac{1}{R_2C_2}\right) = \frac{2}{R_2C_1} = \frac{\omega_p}{Q_p}$$

Use the above two to obtain:

$$|K| = 2Q_p^2 \quad \text{(Eq. 8)}$$

This last relation indicates that the gain and the quality factor are related, and can't be set independently.

Design Procedure 1: Choose C first

1. Choose a capacitor value C based on practical considerations (self-resonance frequency, size, cost, etc.)
2. Calculate R_1 using the design specifications and capacitor value: $R_1 = \frac{1}{4\pi f_0 Q_p} \times \frac{1}{C}$
3. Calculate R_2 using $R_2 = 2|K|R_1$

Design Procedure 2: Choose R_1 first

1. Choose R_1 based on practical considerations (must be large enough to not significantly load previous stage, etc.)
2. Calculate R_2 using $R_2 = 2|K|R_1$
3. Calculate C using $C = \frac{1}{4\pi f_0 Q_p} \times \frac{1}{R_1}$

The same practical considerations for capacitor and resistor values apply to both procedures. For example, if the calculated value of R_1 is too small using procedure 1, then a new capacitor value should be chosen.